Método de Runge-Kutta para Sistemas de Ecuaciones

Dada dos ecuaciones diferenciales de primer orden:

$$U_1^{'} = f(t, U_1, U_2)$$

$$U_2^{'} = g(t, U_1, U_2)$$

Con sus condiciones iníciales (n=0):

$$U_1(t_o) = \alpha$$

$$U_2(t_o) = \beta$$

Método RK2

Calculo de las K's para la integración n+1

$$K1 = h \begin{Bmatrix} f\big(t_n, U_1(t_n), U_2(t_n)\big) \\ g\big(t_n, U_1(t_n), U_2(t_n)\big) \end{Bmatrix}$$

$$K2 = h \begin{Bmatrix} f\big(t_n + h, U_1(t_n) + K_{1,U1}, U_2(t_n) + K_{1,U2}\big) \\ g\big(t_n + h, U_1(t_n) + K_{1,U1}, U_2(t_n) + K_{1,U2}\big) \end{Bmatrix}$$

$$U_1(t_{n+1}) = U_1(t_n) + \frac{1}{2}\big(K_{1,U1} + K_{2,U1}\big)$$

$$U_2(t_{n+1}) = U_2(t_n) + \frac{1}{2}\big(K_{1,U2} + K_{2,U2}\big)$$

Método RK4

Calculo de las K's para la integración n+1

$$K1 = h\begin{cases} f\big(t_n, U_1(t_n), U_2(t_n)\big) \\ g\big(t_n, U_1(t_n), U_2(t_n)\big) \end{cases}$$

$$K2 = h\begin{cases} f\left(t_n + \frac{h}{2}, U_1(t_n) + \frac{K_{1,U1}}{2}, U_2(t_n) + \frac{K_{1,U2}}{2}\right) \\ g\left(t_n + \frac{h}{2}, U_1(t_n) + \frac{K_{1,U1}}{2}, U_2(t_n) + \frac{K_{1,U2}}{2}\right) \end{cases}$$

$$K3 = h\begin{cases} f\left(t_n + \frac{h}{2}, U_1(t_n) + \frac{K_{2,U1}}{2}, U_2(t_n) + \frac{K_{2,U2}}{2}\right) \\ g\left(t_n + \frac{h}{2}, U_1(t_n) + \frac{K_{2,U1}}{2}, U_2(t_n) + \frac{K_{2,U2}}{2}\right) \end{cases}$$

$$K4 = h\begin{cases} f\big(t_n + h, U_1(t_n) + K_{3,U1}, U_2(t_n) + K_{3,U2}\big) \\ g\big(t_n + h, U_1(t_n) + K_{3,U1}, U_2(t_n) + K_{3,U2}\big) \end{cases}$$

$$U_1(t_{n+1}) = U_1(t_n) + \frac{1}{6}\big(K_{1,U1} + 2K_{2,U1} + 2K_{3,U1} + K_{4,U1}\big)$$

$$U_2(t_{n+1}) = U_2(t_n) + \frac{1}{6}\big(K_{1,U2} + 2K_{2,U2} + 2K_{3,U2} + K_{4,U2}\big)$$